Reminiscences of my work with Richard Lewis Arnowitt

Pran Nath*
Department of Physics, Northeastern University, Boston, MA 02115, USA

This article contains reminiscences of the collaborative work that Richard Arnowitt and I did together which stretched over many years and encompasses several areas of particle theory. The article is an extended version of my talk at the Memorial Symposium in honor of Richard Arnowitt at Texas A&M, College Station, Texas, September 19-20, 2014.

My collaboration with Richard Arnowitt (1928-2014)[1] started soon after I arrived at Northeastern University in 1966 and continued for many years. In this brief article on the reminiscences of my work with Dick I review some of the more salient parts of our collaborative work which includes effective Lagrangians, current algebra, scale invariance and its breakdown, the $U(1)$ problem, formulation of the first local supersymmetry and the development of supergravity grand unification in collaboration of Ali Chamseddine, and its applications to the search for supersymmetry.

I. EFFECTIVE LAGRANGIANS AND CURRENT ALGEBRA

In 1964 Gell-Mann [1] proposed that "quark-type" equal-time commutation relations for the vector and the axial vector currents of weak interaction theory serve as a basis for calculations involving strongly interacting particles. Combined with the conserved vector current (CVC), partially conserved vector current (PCAC) and the soft pion approximation many successful results were obtained (see, for example, [2]). However, around 1967 an important issue arose which concerned the breakdown of the soft pion approximation in the analysis of $\rho \to \pi\pi$ and $A_1 \to \rho\pi$ where the soft pion approximation gave very poor results [3, 4]. This problem was overcome in work with Dick and Marvin Friedman by giving up the soft pion approximation and using the effective Lagrangian method to compute the mesonic vertices [5–7]. A number of other techniques were being pursued at that time such as Ward identities by Schnitzer and Weinberg [8], phenomenological Lagrangians by Schwinger[9], Wess and Zumino [10], and by Ben Lee and Nieh [11] and other techniques [12–14].

Here I describe briefly the approach that Dick Arnowitt, Marvin Friedman and I followed which first of all involved developing an effective Lagrangian for the $\pi\rho A_1$ system but then using current algebra conditions to constrain the parameters of the effective Lagrangian. The effective Lagrangian was a deduction from the following set of conditions: (i) single particle saturation in computation of T-products of currents, (ii) Lorentz invariance, (iii) "spectator" approximation, and (iv) locality which implies a smoothness assumption on the vertices. The above assumptions lead us to the conclusion that the simplest way to achieve these constraints is via an effective Lagrangian which for T-products of three currents requires writing cubic interactions involving π, ρ and A_1 fields and allowing for no derivatives in the first -order formalism and up to one-derivative in the second order formalism. The effective Lagrangian is to be used to first order in the coupling constants for three point functions [5–7] and to second order in the coupling constants for 4-point functions and to $N-2$ order in the coupling constant for N point functions. The second step consisted of the imposition of the constraints of current algebra, CVC and PCAC to determine the parameters appearing in the effective Lagrangian. Thus in addition to [5–7] several other applications of the effective Lagrangian method were made [15–21]. Below we give some further details of the effective Lagrangian construction.

We begin by considering a T-product of three currents, i.e.

$$F^{\alpha\mu\beta} \equiv < 0|T(A^\alpha_a(x)V^\mu_c(z)A^\beta_b(y))|0>, \tag{1}$$

For the time ordering $x^0 > z^0 > y^0$ Eq. (1) can be expanded so that

$$F^{\alpha\mu\beta} = \sum_{n,m} < 0|A^\alpha_a|n >< n|V^\mu_c|m >< m|A^\beta_b|0> . \tag{2}$$

Here the states n and m can only be either π or A_1 mesons. It is clear that the matrix elements that are involved are $< 0|A^\alpha_a|\pi q_1 a_1 >$, $< \pi q_1 a_1|V^\mu_c|\pi q_2 a_2 >$ and $< \pi q_2 a_2|A^\beta_b|0 >$ and additional terms where π is replaced by A_1. For another time ordering , i.e., $y^0 > x^0 > z^0$ one has

$$F^{\alpha\mu\beta} = \sum_{n,m} < 0|A^\beta_b|n >< n|A^\alpha_a|m >< m|V^\mu_c|0 >, \tag{3}$$

where the state n must be a π or A_1 state. However, the state m must be a ρ state to have non-vanishing matrix elements. In addition to the above we must also include two particle intermediate states so that for the time ordering $x^0 > y^0 > z^0$ we have

* Email: nath@neu.edu
[1] Obituary: Richard Lewis Arnowitt, Physics Today 67(12), 68 (2014). Submitted by Stanley Deser (Brandeis and Caltech), Charles Misner (University of Maryland), Pran Nath (Northeastern University), and Marlan Scully (Texas A&M and Princeton University).

$$F^{\alpha\mu\beta} = \sum_{n,m,k} < 0|A_a^\alpha|n,m >< n,m|V_c^\mu|k >< k|A_b^\beta|0 > \ . \tag{4}$$

Here, for example, n must be either π or A_1, and m must be a ρ and k a π or A_1. This means that one of the particles in the matrix element $< n,m|V^\nu|k >$ must be a "spectator". Thus one has

$$< \pi q_1 a_1 \rho, p_1 a_3|V_c^\mu|\pi q_2 a_2 > = \delta_{a_1 a_2}\delta^3(\vec{q}_1 - \vec{q}_2) \times < \rho p_1 a_3|V_c^\mu|0 > \ . \tag{5}$$

Such contributions are required by Lorentz invariance and crossing symmetry. Indeed Eq. (5) is the cross diagram contribution to Eq. (3). Now the vacuum to one particle matrix elements of currents define the interpolating constants F_π, g_A, g_ρ so that one has $< 0|A_a^\alpha(0)|\pi q_1 a_1 >= F_\pi q^\alpha$, $< 0|A^\alpha(0)|A_1\sigma >= g_A\epsilon^{\alpha\sigma}$ and $< 0|V^\mu(0)|\rho\sigma >= g_\rho\epsilon^{\mu\sigma}$ (where we have suppressed the normalizations and iso-spin factors). These results can be simulated by writing $V_a^\mu = g_\rho\tilde{v}_a^\mu$ and $A_a^\mu = g_A\tilde{a}_a^\mu + F_\pi\partial^\mu\tilde{\phi}_a$ where the tilde fields are the in-fields. For the matrix elements of a current between two particle states, e.g., $< \pi q_1 a|V_c^\mu(0)|\pi q_2 b >$ we write (suppressing normalization factors) $< \pi q_1 a|V_c^\mu(0)|\pi q_2 b >= \epsilon_{abc}\Delta_\lambda^\mu\Gamma^\lambda(q_1,q_2)$ where Δ_λ^μ is a ρ propagator and the vertex Γ^λ can be expanded in a power series

$$\Gamma^\lambda(q_1,q_2) = (q_1^\lambda + q_2^\lambda)(\alpha_1 + \alpha_2 k^2 + \cdots)\,, \tag{6}$$

where $k^\lambda = q_1^\lambda - q_2^\lambda$. Now the two particle matrix element of V^μ, i.e., $< \pi q_1 a|V_c^\mu(0)|\pi q_2 b >$, can be obtained if we replace the v_c^μ field in terms of the bilinear product of two pion in-fields. Including the vacuum to one particle matrix element contribution this leads us to a following form for V_c^μ.

$$V_c^\mu(x) = g_\rho\tilde{v}_c^\mu + \epsilon_{abc}\int d^4y\Delta_\lambda^\mu(x-y) \times \left[\alpha_1 - \alpha_2\Box^2 + \cdots\right]\tilde{\phi}_a(y)\partial^\lambda\tilde{\phi}_b(y) + \cdots\,, \tag{7}$$

where the dots at the end stand for other bilinear terms that are left out. The form above guarantees crossing symmetry. A very similar analysis holds for the axial current. For further analysis it is useful to replace the in-fields by the Heisenberg field operators which obey the Heisenberg field equations. Thus we consider the ρ field v_c^μ to obey the Heisenberg field equation

$$K_\lambda^\mu(x)v_c^\lambda(x) = g_\rho^{-1}\epsilon_{abc}\left[\alpha_1 - \alpha_2\Box^2 + \cdots\right] \times \phi_a(x)\partial^\mu\phi_b(x) + \cdots\,, \tag{8}$$

where K_λ^μ is the Proca operator $K_\lambda^\mu = (-\Box^2 + m_\rho^2)\delta_\lambda^\mu + \partial^\mu\partial_\lambda]$. Thus Eq. (7) is now equivalent to the relation

$$V_a^\mu(x) = g_\rho v_a^\mu(x). \tag{9}$$

In a similar way one has

$$A_a^\mu(x) = g_A a_a^\mu(x) + F_\pi\partial^\mu\phi_a(x)\,, \tag{10}$$

It should be clear that Eqs. (9) and (10) are a consequence of single particle saturation assumption and not meant to be fundamental postulates. Thus our approach differs from the one by Lee, Weinberg and Zumino [22]. It should now be noted that Eqs. (9) and (10) are to be used in the following way: we solve the Heisenberg equations and then use them to first order in the coupling constant in the computation of T-product involving three currents. This is equivalent to the in-field expansion of Eq. (7). However, due to the presence of the propagators Δ_λ^μ etc the locality of $V_c^\mu(x)$ etc is not guaranteed. Thus if we demand that $v_a^\mu(x)$, $\pi_a(x)$ and $a_a^\mu(x)$ be local field operators, whose commutators for space-like separations vanish, then this condition can be guaranteed if we require that the sources for the fields v_c^μ etc arise from a Lagrangian with interactions that are cubic in the fields. Thus the arguments laid out above lead us to an effective Lagrangian of the form

$$\mathcal{L}_{eff}^{(3)} = \mathcal{L}_0 + g\mathcal{L}_3 \tag{11}$$

where $\mathcal{L}_0 = \mathcal{L}_{0\pi} + \mathcal{L}_{0\rho} + \mathcal{L}_{0A_1}$. We note that we arrived at Eq. (11) purely from the conditions of (i) single particle saturation of T -products, (ii) spectator approximation, (iii) Lorentz invariance and crossing symmetry and , (iv) locality. The current algebra constraints, i.e., current algebra commutation relations, CVC and PCAC have played no role in the analysis thus far.

The analysis above can be extended to higher point functions. For instance, for the computation of the four -point function,

$$F^{\alpha\beta\mu\nu}(x,y,z,w) \equiv < T(A^\alpha(x)A^\beta(y)V^\mu(z)V^\nu(\omega)) >\,, \tag{12}$$

the implementation of the single particle saturation requires that we include the following set of contributions, and (i) diagrams where we have a cascade of three point vertices, and (ii) diagrams where we have four point vertices. The diagrams of type (i) arise from $\mathcal{L}_3$, and diagrams of type (ii) arise from $\mathcal{L}_4$. The Lagrangian

$$\mathcal{L}_{eff}^{(4)} = \mathcal{L}_0 + g\mathcal{L}_3 + g^2\mathcal{L}_4\ , \tag{13}$$

is to be used to the first non-vanishing order, i.e., order g^2, discarding disconnected diagrams to compute the T product of four currents. It is now straightforward to extend the analysis to the computation of T products of N-point functions. Thus using the same principles as above, the analysis of N-point functions involves using an interaction Lagrangian [15, 16]

$$\mathcal{L}_{eff}^{(N)} = \mathcal{L}_0 + g\mathcal{L}_3 + g^2\mathcal{L}_4 + \cdots + g^{k-2}\mathcal{L}_k + \cdots + g^{N-2}\mathcal{L}_N\ . \tag{14}$$

The coupling in $\mathcal{L}_k$ will be viewed as $\mathcal{O}(g^{k-2})$. Further, the computation of an N -point function is then done using the effective Lagrangian to order $N-2$ in the couplings. The effective Lagrangian techniques described above have a much larger domain of validity than the specific example of the mesonic system being discussed here.

Current Algebra Constraints: After ensuring that the constraints of single particles saturation, spectator approximation, Lorentz invariance and locality can be embodied by writing an effective Lagrangian we impose constraints of current algebra. As mentioned earlier these consist of (i) equal-time commutation relations on the densities[2], (ii) CVC, and (iii) PCAC. The $\pi-\rho-A_1$ effective Lagrangian allowed one to compute $\pi\rho A_1$ processes without the soft pion approximation and get results consistent with data. As discussed above the technique of effective Lagrangian allows one to obtain Lagrangians obeying current algebra constraints for higher points functions. Thus for $SU(2)\times SU(2)$ current algebra constraints the effective Lagrangian method was used not only for the processes $\rho\to\pi\pi$, $A_1\to\pi\rho$ but also to give the first analysis of $\pi\pi\to\pi\pi$ scattering using hard pion current algebra [17]. The effective Lagrangian method was then extended to include $SU(3)\times SU(3)$ current algebra constraints which allowed an analysis of the $K\ell_3$ and πK scattering [18–20, 23]. In a later work [23] the current algebra constraints were also applied to the Veneziano model.

Effective vs Phenomenological Lagrangians: The effective Lagrangian technique used in [5–7, 15–21]. is different from the works of Schwinger [9], Wess and Zumino [10], and of Ben Lee and Nieh [11] which were phenomenological Lagrangians. In phenomenological Lagrangian, one starts by constructing Lagrangians which have $SU(2)\times SU(2)$ or $SU(3)\times SU(3)$ invariance. The invariance is then broken by additional terms which are introduced by hand. In the effective Lagrangian approach no a priori assumption was made regarding the type of symmetry breaking, chiral or ordinary. The current algebra constraints alone determine the nature of symmetry breaking. What we found was that for hard meson current algebra with single meson dominance of the currents and of the σ commutator, the chiral symmetry breaking was broken only by [16] $(3,3^*)+(3^*,3)$. This type of breaking had been proposed by Gell-Mann, Oakes and Renner [24].

The Axial Current Anomaly: Beginning in 1967 one of the big puzzles related to the Veltman theorem [25] which was that in the soft pion approximation the pion decay into two $\gamma' s$ vanished. It was generally held that a possible source of this problem could be that the soft pion approximation was breaking down due to a very rapid variation of the matrix elements as we went off the pion mass-shell. However, in a paper in 1968 we discovered [26] that hard pion analysis also gave a vanishing $\pi^0\to 2\gamma$ decay. This lead us to propose a modification of the PCAC condition by introducing an axial current anomaly which exists even in the chiral limit [26], i.e., we proposed $\partial_\mu A^\mu_a = F_a m_a^2\phi_a + \lambda d_{abc}\epsilon_{\mu\nu\alpha\beta}F_b^{\mu\nu}F_c^{\alpha\beta} + \lambda'\epsilon_{\mu\nu\alpha\beta}F_a^{\mu\nu}\phi^{\alpha\beta}$, where $a,b,c = 1\cdots 8$. With the above modification a number of other decays were also computed such as $\eta\to 2\gamma$ and $\rho^0\pi^0\gamma$. The axial anomaly was simultaneously computed from triangle loops with fermions by Bell and Jackiw [27] and by Adler [28].

Scale Invariance and scale breaking: The scaling [29] observed in electro-production data lead to the hypothesis that physical laws are scale-invariant at high energies [30–32]. While such a hypothesis may hold at high energy, it is certainly violated at low and intermediate energies as physics is not scale-invariant there. Thus at low and intermediate scales dimensioned parameters such as masses appear. Supposing that scale invariance is of fundamental significance then it is of relevance to ask the manner of its breakdown. Several works had tried to approach this problem in the early seventies [33–36]. In our analysis of scale invariance and its breakdown the stress tensor and its trace play a central role. Out approach to scale invariance and its breakdown was parallel to our approach to current algebra [37–39]. Thus in the analysis of the current algebra constraints we assumed that the vector current was dominated by a vector spin 1 particle and the axial current by an axial vector and a pseudoscalar meson. In an analogous fashion we assumed that the stress tensor was dominated by $J^P = 2^+, 0^+$ mesons in a new field current identity. Applications were made in the deduction of light-cone algebra, deep-inelastic scattering [39] and e^+e^- annihilation at intermediate energies [38].

II. THE $U(1)$ PROBLEM

The $U(1)$ problem relates to the fact that the ordinary $U(3)\times U(3)$ current algebra leads to the ninth pseudoscalar meson being light [40, 41]: $m_{\eta'} < \sqrt{3}m_\pi$. Formally a resolution was proposed by t'Hooft [42] who showed that the instanton solution to the Yang-Mills theory provides a contribution to the η' mass. However, the t'Hooft solution is inadequate as it does not make contact with the quark-antiquark annihilation of the singlet pseudo scalar into gluons [43]. Further, Witten[44] showed that a resolution of the $U(1)$ anomaly arises in the $1/N$ expansion of QCD. Thus the η' is massless in the $N\to\infty$ limit but significant non-zero contributions arise from terms which are $1/N$ smaller than the leading terms and split η' from the octet. We examined the problem from an effective Lagrangian view point. We introduced a Kogut-Susskind ghost field K^μ and constructed a closed

[2] The imposition of the absence of q-number Schwinger term gives the first Weinberg Sum rule: $g_\rho^2/m_\rho^2 = g_A^2/m_A^2 + F_\pi^2$.

form solution for the effective Lagrangian[45–48].

$$\mathcal{L} = \mathcal{L}_{CA} + \frac{1}{2C}(\partial_\mu K_\nu)^2 + G\partial_\mu K^\mu - \theta\partial_\mu K^\mu \ , \quad (15)$$

which gives a complete description of the interaction of the field K^μ with the mesonic fields. Thus G is a function which depends on the spin zero and spin one mesonic fields, θ is the strong CP violating parameter of QCD, and C is the strength of the topological charge $i < T(K_\mu K_\nu) > = -C\eta_{\mu\nu}/q^2 + \cdots$. Ignoring spin one fields, G is determined to be of the form given by Rosenzweig et.al. [49], $G = \frac{1}{2}\left[\text{lndet}\xi - \text{lndet}\xi^\dagger\right]$, where $\xi = (u_a + iv_a)\lambda_a$ and where u_a and v_a are scalar and pseudo-scalar densities. Using the effective Lagrangian which includes the effect of the $U(1)$ anomaly, we found a sum rule of the form [46, 47] [3]

$$(F_{88} + \sqrt{2}F_{98})^2 m_\eta^2 + (F_{89} + \sqrt{2}F_{99})^2 m_{\eta'}^2 = 3m_\pi^2 F_\pi^2 + \frac{4}{3}N_f^2 \left(\frac{d^2E}{d\theta^2}\right)_{\theta=0}^{N_f=0} , \quad (16)$$

where N_f is the number of light quark flavors. If one ignores the first and the last terms, sets $F_{89} = 0$, and let $F_{99} \to \sqrt{N_f/6}F_\pi$ ($N_f = 3$), one finds the Weinberg result [41]

$$m_{\eta'} < \sqrt{3}m_\pi \, . \quad (17)$$

Further, in the limit $m_\pi = 0 = m_\eta$, $F_{89} = 0$ and $F_{99} \to \sqrt{N_f/6}F_\pi$ one finds Witten's result [44]

$$m_{\eta'}^2 \to \frac{4N_f}{F_\pi^2}\left(\frac{d^2E(\theta)}{d\theta^2}\right)_{\theta=0}^{N_\ell=0} . \quad (18)$$

In addition to the work of [45–49] a Lagrangian formulation including the $U(1)$ axial anomaly was given by Di-Vecchia and Veneziano [50]. Witten [51] has shown that these Lagrangian formulations which include the effect of the $U(1)$ anomaly and solve the η' puzzle are consistent with the large N chiral dynamics.

III. LOCAL SUPERSYMMETRY

It was in 1974 when I was at the XVII ICHEP Conference in London that I first became interested in supersymmetry. On my return to Boston I talked to Dick to work in this area. At that time SUSY was a global symmetry, and we thought that if it is a fundamental symmetry it ought to be a local symmetry. Very quickly we realized that *gauging of supersymmetry requires bringing in gravity*, and we thought that the direct course of action was to extend the geometry of Einstein gravity to superspace geometry. This lead to the formulation of gauge supersymmetry [52] based on a single tensor superfield $g_{\Lambda\Pi}(z)$ in superspace consisting of bose and fermi co-ordinates, i.e., $z^\Lambda = (x^\mu, \theta^{\alpha i})$ where x^μ are the bose co-ordinates of ordinary space-time and $\theta^{\alpha i}$ are anti-commuting fermi co-ordinates. We considered a line element of the form $ds^2 = dz^\Lambda g_{\Lambda\Pi}(z)dz^\Pi$ and required its invariance under the general co-ordinate transformations in superspace $z^\Lambda = z'^\Lambda + \xi^\Lambda(z)$ which leads to the transformations of the superspace metric tensor of the form

$$\delta g_{\Lambda\Pi}(z) = g_{\Lambda\Sigma}\xi^\Sigma_{,\Pi} + (-1)^{\Lambda+\Lambda\Sigma}\xi^\Sigma_{,\Lambda}\xi_{\Sigma\Pi} + g_{\Lambda\Pi,\Sigma}\xi^\Sigma \, , \quad (19)$$

where $(-1)^\Lambda = 1(-1)$ when Λ is bosonic (fermionic) etc. One may also introduce a supervierbein so that

$$g_{\Lambda\Pi}(z) = V^A{}_\Lambda(z)\eta_{AB}(-1)^{(1+B)\Pi}V^B{}_\Pi(z) \quad (20)$$

where η_{AB} is a tangent space metric so that

$$\eta_{AB} = \begin{pmatrix} \eta_{mn} & 0 \\ 0 & k\eta_{ab} \end{pmatrix} \quad (21)$$

where η_{mn} is the metric in bose space and η_{ab} is the metric in fermi space, so that $\eta_{ab} = -(C^{-1})_{ab}$ where C is the charge conjugation matrix. In Eq. (21) k is an arbitrary parameter. Global supersymmetry transformations are generated by ξ^Λ of the form

$$\xi^\mu = i\bar{\lambda}\gamma^\mu\theta, \quad \xi^\alpha = \lambda^\alpha \, , \quad (22)$$

where λ^α are constant infinitesimal anti-commuting parameters. For the global supersymmetry case the metric that keeps the line element invariant is given by

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} \, , \\ g_{\mu\alpha} &= -i(\bar{\theta}\gamma^m)_\alpha\eta_{m\mu} \, , \\ g_{\alpha\beta} &= k\eta_{\alpha\beta} + (\bar{\theta}\gamma_m)_\alpha(\bar{\theta}\gamma^m)_\beta \, . \end{aligned} \quad (23)$$

To set up an action principle in superspace required defining a superdeterminant. In work with Bruno Zumino [53] it was shown that for a matrix M_{AB} with bosonic and fermionic components $\{M_{\mu\nu}, M_{\mu\alpha}, M_{\alpha\mu}, M_{\alpha\beta}\}$ where $M_{\mu\nu}$ and $M_{\alpha\beta}$ are bosonic and $M_{\mu\alpha}$ and $M_{\alpha\mu}$ are fermionic quantities, $det(M)$ is given by

$$detM = (detM_{\mu\nu})det(M^{-1})^{\alpha\beta} \, . \quad (24)$$

This also then gives $\sqrt{(-g)} = [-(detg_{\mu\nu})(detg^{\alpha\beta})]^{1/2}$. Using the above one can then set up an action principle in superspace so that

[3] F_{ab} are defined through divergence of the axial current so that

$$\partial_\mu A_a^\mu \supset F_{ab}\mu_{bc}\chi_c + \sqrt{\frac{2}{3}}N_\ell\delta_{a9}\partial_\mu K^\mu.$$

$$A = \int d^8z\sqrt{-g}R\,, \tag{25}$$

where R is a superspace curvature scalar defined by $R = (-1)^a g^{AB}R_{BA}$. A more general action than the one in Eq. (25) can be gotten by including an additional term, i.e., $2\lambda\int d^8z\sqrt{-g}$, which leads to the field equations in superspace to read $R_{AB} = \lambda g_{AB}$. Some very encouraging features emerged. From the superspace transformations we were able to recover both the Einstein gauge invariance and the Yang-Mills gauge invariance which appeared to be quite remarkable. The metric superfield g_{AB} contains many fields. Thus $g_{\mu\nu}(z) = g_{\mu\nu}(x)+\cdots$, $g_{\mu\alpha}(z) = \bar\psi_{\mu\alpha}(x)+(\bar\theta M_x)_\alpha B^x_\mu+\cdots$, $g_{\alpha\beta}(z) = (\eta F(x))_{\alpha\beta} + (\bar\theta M_x)_{[\alpha}\bar\chi^x_{\beta]}(x) + \cdots$. Here $g_{\mu\nu}$ is the metric in ordinary bose space and contains the spin 2 graviton field, $\psi_{\mu\alpha}$ is a spin 3/2 field, B_μ is a vector field and $F(x)$ and χ_β were spin 0 and spin 1/2 fields. The implications of this theory was examined in several works [54–61].

Supergravity and Gauge Supersymmetry: While gauge supersymmetry was the first realization of a local supersymmetry, its field content is rather complicated. Further developments in this field occurred via formulation of local supersymmetry involving just the spin 2 and spin 3/2 fields [62, 63] (for a review see [64]), i.e., supergravity. The question then is what is the connection of gauge supersymmetry to supergravity. In [65–67] we showed that supergravity can be recovered from gauge supersymmetry if one discards all other fields in the metric $g_{AB}(z)$ except the spin 2 and spin 3/2 fields and considers the $k \to 0$ (where k is defined in Eq. (23)) limit of gauge supersymmetry.

Thus to recover supergravity from gauge supersymmetry we construct the metric only in terms of spin 2 and spin 3/2 fields. Specifically we want to construct g_{AB} depending on the fields $e^m_\mu(x)$ and $\psi^\alpha_\mu(x)$ and find ξ^A so that the transformation equation Eq. (19) leads correctly to the supergravity transformations for $e^m_\mu(x)$ and $\psi_\mu(x)$ so that

$$\begin{aligned}\delta e^m_\mu &= i\bar\psi_\mu(x)\gamma^m\lambda(x)\,,\\ \frac{1}{2}\delta\psi_\mu(x) &= (\partial_\mu + \Gamma_\mu)\lambda(x),,\end{aligned} \tag{26}$$

where $\lambda(x)$ are the transformation parameters and Γ_μ is to be determined. This is to be done by using Eq. (19) order by order in θ by the process of gauge completion developed in [65–67]. Using this procedure one finds [65–67]

$$\begin{aligned}\xi^\mu(z) &= i\bar\lambda(x)\gamma^\mu\theta + \frac{1}{2}(\bar\psi_m\gamma^\mu\theta)(\bar\lambda\gamma^m\theta) + \Delta\xi^\mu(z)\,,\\ \xi^\alpha(z) &= \lambda^\alpha(x) - \frac{i}{2}\psi_m{}^\alpha\bar\lambda\gamma^m\theta - \frac{1}{4}\psi_r{}^\alpha(\bar\psi_m\gamma^r\theta)(\bar\lambda\gamma^m\theta)\\ &\quad - i(\Gamma_m\theta)^\alpha(\bar\lambda\gamma^m\theta) + \Delta\xi^\alpha(z)\,,\\ g_{\mu\nu}(z) &= g_{\mu\nu}(x) + i\bar\psi_{(\mu}\gamma_{\nu)}\theta - i\bar\theta\gamma_{(\mu}\Gamma_{\nu)}\theta\\ &\quad - (\bar\psi_\mu\gamma^m\theta)(\bar\psi_\nu\gamma_m\theta) + \delta g_{\mu\nu}\,,\\ g_{\mu\alpha}(z) &= -i(\bar\theta\gamma_m)_\alpha e^m_\mu + (\bar\psi^\mu\gamma_m\theta)(\bar\theta\gamma^m)_\alpha + \Delta g_{\mu\alpha}\,,\\ g_{\alpha\beta}(z) &= k\eta_{\alpha\beta} + (\bar\theta\gamma_m)_\alpha(\bar\theta\gamma^m)_\beta + \Delta g_{\alpha\beta}\,.\end{aligned} \tag{27}$$

Here $\Delta\xi^\mu, \Delta\xi^\alpha$ etc are quantities that depend on k and also contain terms $O(\theta^3)$ and higher. An important result that emerges is that the gauge completion procedure using Eq. (19) determines the vierbein affinity to be that of supergravity, i.e.,

$$\Gamma_\mu = \frac{i}{4}\sigma_{rs}\omega_\mu{}^{rs}\,, \tag{28}$$

where $\omega_\mu{}^{rs}$ correctly includes the supergravity torsion. In the $k \to 0$ Eq. (27) give the correct supergravity transformation equations as well as the correct $g_{\Lambda\Pi}$ up to $\mathcal{O}(\theta^2)$. Further the dynamical equations of gauge supersymmetry $R_{\Lambda\Pi} = 0$ (setting $\lambda = 0$) produce correctly the dynamical equations of supergravity. We note here that the integration of Eq. (19) beyond linear order in θ requires use of on-shell constraints, i.e., they can be integrated if we impose field equations. Integration off the mass-shell requires Breitenlohner fields [68] which allows gauge completion without use of field equations [66].

Geometrically the connection between gauge supersymmetry and supergravity is the following: Gauge supersymmetry is a geometry with the tangent space group $OSp(3, 1|4N)$ while the tangent space group of supergravity geometry is $O(3, 1) \times O(N)$. In the limit $k \to 0$ the geometry of gauge supersymmetry contracts to the supergravity geometry. The contraction produces the desired torsions [67] needed in the superspace formulation of supergravity [69–71] and the tangent space group $OSp(3, 1|4N)$ of gauge supersymmetry reduces [67] to the tangent space group $O(3, 1) \times O(N)$ of supergravity. Thus the the $k \to 0$ limit of the geometry of gauge supersymmetry correctly produces the supergravity geometry in superspace.

IV. GRAVITY MEDIATED BREAKING AND SUPERGRAVITY GRAND UNIFICATION

This phase of research with Dick involves Ali Chamseddine. In 1980 I was on sabbatical leave at CERN and it was sheer good luck that I met Ali there. Ali was aware of the work that Dick and I had done on local supersymmetry since at the suggestion of his thesis advisor Abdus Salam, he had worked on gauge supersymmetry which appears as a part of his Ph.D. thesis at Imperial College, London [72, 73]. Beyond that he had worked on supergravity as a gauge theory of supersymmetry [74]. At the time I met Ali, Dick and I were looking for a research associate on our NSF grant and we thought that Ali would be a good fit for us because of our common interests in supersymmetry and so after a conversation with Dick, I made an offer to Ali to visit Boston after culmination of his Fellowship period at CERN. Ali arrived in Boston in January of 1981. He had recently

finished a work on the coupling of $N = 4$ supergravity to $N = 4$ matter [75] and was actively working on interacting supergravity in ten-dimensions and its compactification to a four -dimensional theory. After his work on 10-dimensional supergravity was finished in the beginning of the Fall 1981 [76], our interests converged on model building within $N = 1$ supergravity framework. $N = 1$ supersymmetry has been shown to have many desirable properties including the fact that it provided a technical solution to the gauge hierarchy problem. However, at that time there were no acceptable supersymmetry based particle physics models where one could break supersymmetry spontaneously in a phenomenologically viable way.

As mentioned our analysis started in the beginning of the Fall of 1981. At that time only the most general coupling of one chiral field with supergravity was known through the 1979 work of Cremmer etal [77]. However, the construction of a particle physics model required extension to an arbitrary number of chiral fields. Thus the first task was to construct a Lagrangian with an arbitrary number of chiral fields which couple to an adjoint representation of a gauge group and to $N = 1$ supergravity. This analysis was rather elaborate and took us up to the early spring of 1982 to complete. The Lagrangian showed some very interesting features in that there were terms in the scalar potential which were both positive and negative and thus an opportunity existed of their cancellation after spontaneous breaking of supersymmetry which was a very desirable feature for the generation of a viable model. Although we had the couplings in the early spring of 1982, we did not publish them immediately since we were after construction of a realistic supergravity grand unified model. The $N = 1$ supergravity couplings were published later in the Trieste Lecture Series titled *"Applied N= 1 Supergravity"* [78]. Our analyses to be published later in the Summer and Fall of 1982 [79–81] were based on the supergravity couplings contained in [78]. (The supergravity couplings with an arbitrary number of chiral fields were independently obtained by Cremmer etal [82, 83] and also by Bagger and Witten [84]). In our attempt to construct the supergravity grand unified model one of the phenomena we noticed concerned the lifting of the degeneracy after spontaneous breaking of the GUT symmetry. Thus in a globally supersymmetric theory the breaking of $SU(5)$ leads to three possible vacua which have the vacuum symmetry given by $SU(5)$, $SU(4) \times U(1)$ and $SU(3) \times SU(2) \times U(1)$ which are degenerate. For the supergravity case we found that this degeneracy was lifted by gravitational interactions. This phenomena was also observed by Weinberg [85].

However, to construct a realistic grand unified model we needed to break the $N = 1$ supersymmetry and grow mass terms for the squarks and the sleptons which were large enough to have escaped detection in current experiment. For the breaking of supersymmetry we utilized the superHiggs effect where the superHiggs field develops a vacuum expectation value which is $\mathcal{O}(\mathrm{M_{Pl}})$. However, a superHiggs field could not be allowed to interact with the matter fields in the superpotential directly as that would lead to Planck size masses for the matter fields. For this reason it was necessary to create two sectors, one where only quarks, leptons and Higgs fields reside and the other sector where the superHiggs field resides. In this case the only coupling that exists between the observed or the visible sector and the superHiggs or the hidden sector was through gravitational interactions. In this way we could generate soft terms in the visible sector which could be of the electroweak size. Thus we assumed the superpotential to be of the form $W = W_1 + W_2$ where W_1 contains only matter and Higgs fields and W_2 only the superHiggs field z. Assuming $W_2 = m^2 f(z)$ one finds that the soft terms of size $\mathcal{O}(m^2/M_{\rm Pl})$ grow in the visible sector after spontaneous breaking of supersymmetry in the hidden sector. Thus if m is of size the intermediate scale, i.e., $\mathcal{O}(10^{11})$ GeV, one finds that soft terms in the visible sector are size the electroweak scale. The intermediate scale of $\mathcal{O}(10^{11})$ GeV could arise from a strongly interacting gauge group in the hidden sector.

The mechanism discussed above avoids the appearance of Planck size masses in the soft sector. However, since we were working with a grand unified theory where heavy GUT fields with masses of size $\mathcal{O}(M_G)$ appear, it was possible that the soft terms could be size $\mathcal{O}(M_G)$. Our analysis of [79, 80] (see also [86]) showed that the soft terms are indeed independent of M_G. In the analysis of [79] the generation of soft terms was also shown to lead to the breaking of the electroweak symmetry resolving a long standing problem of the Standard Model where the breaking is induced by the assumption of a tachyonic mass term for the Higgs boson. The preceding discussion shows that our efforts were successful and we were able to formulate the first phenomenologically viable supergravity grand unified model with gravity mediated breaking [79–81] which lead to several further works involving Ali, Dick, and myself [78, 80, 87–93]. A history of the development of SUGRA GUT and of these collaborative works are discussed in several reviews [72, 94, 95]. Subsequent to our work of [79] a number of related papers appeared in a short period of time. A partial list of these is given in [96–105].

The 1982 works created a new direction of research where the electroweak physics testable at colliders and in underground experiment could be discussed within the framework of a UV complete model. Specifically testable predictions of supergravity models include electroweak loop corrections to precision parameters such as $g_\mu - 2$, sparticle signatures at colliders, proton decay and dark matter. Several of these topics were worked on in a series of papers involving Ali, Dick and myself [78, 80, 87–93]. Further collaborative work on these topics between Dick and I continued even after Ali left Northeastern University and moved on to work on strings and on non-commutative geometry. Below we give further details of some of the implications of supergravity unified models.

As mentioned earlier a remarkable aspect of supergravity grand unified theory is that soft breaking parameters

can induce breaking of the electroweak symmetry [79] and an attractive mechanism for this is via radiative breaking [97, 99, 101, 104, 106] using renormalization group evolution [107] (for a review see [108]). The radiative electroweak symmetry breaking must, however, be arranged to preserve color and charge conservation [109–111]. The radiative electroweak symmetry breaking can be used to determine the Higgs mixing parameter μ except for its sign. The simplest SUGRA model that emerges is the one with universal soft breaking at the grand unification scale which can be parameterized at the electroweak scale by

$$m_0, m_{1/2}, A_0, \tan\beta, \mathrm{sign}(\mu): \quad \mathrm{mSUGRA} \tag{29}$$

where m_0 is the universal scalar mass, $m_{1/2}$ is the universal gaugino mass, A_0 is the universal trilinear coupling, $\tan\beta = < H_2 > / < H_1 >$ where H_2 gives mass to the up quarks and H_1 gives mass to the down quarks and leptons, and sign(μ) is the sign of μ which is not determined by radiative electroweak symmetry breaking. For more general choices of the Kahler potential and of the gauge kinetic energy function, SUGRA models with non-universalities are obtained [105, 112–114]. Some early work on the signatures of supergravity models can be found in [87–89, 100, 115–122].

Supersymmetric electroweak corrections to $g_\mu - 2$: It was realized early on [90] (see also [123]) that the supersymmetric electro-weak corrections to the anomalous magnetic moment of the muon in supergravity unified models could be of the same size as the electroweak corrections arising from the Standard Model. Specifically it was shown that the supersymmetric electroweak corrections can be substantial for low lying charginos, neutralinos and smuon states circulating in the loops. This result was helpful when the Brookhaven experiment E821 was being conceived. The current data from the Brookhaven experiment E821 [124] which measures $a_\mu = \frac{1}{2}(g_\mu - 2)$ shows a deviation from the Standard Model prediction [125, 126] at the $3\,\sigma$ level, i.e., it gives $\delta a_\mu = (287 \pm 80.) \times 10^{-11}$. It remains to be seen if the observed deviation will survive in future improvement of experiment and also further improvement in the analysis of the hadronic correction.

Supersymmetric signals: Subsequent to the development of the supergravity grand unification, its possible signatures at colliders were investigated. One of the main focus was on jets, leptons and missing energy signals [78, 88, 89]. Initially the analyses were for the on-shell decays of the W and Z boson [78, 88, 89, 116, 127] where an on-shell W decays via the chain $W^\pm \to \chi_1^\pm \chi_2^0$ with the further decays $\chi_1^- \to e^- \bar{\nu} \chi_1^0$ and $\chi_2^0 \to \ell^+ \ell^- \chi_1^0$. This leads to a trileptonic signal $\ell_1 \ell_2 \bar{\ell}_2$ plus missing transverse energy E_T. These on-shell W decay analyses were limited by the constraint $M_{\chi_1^\pm} + m_{\chi_2^0} < M_W$. However, in [128, 129] the analysis was extended to decays when W and Z are off-shell. Here it was shown that strong leptonic signals can arise even when W and Z are off-shell and such signals are now some of the primary modes of discovery for the supersymmetric particles. Leptonic signals were further discussed in several later works [130–132]. A variety of other signatures of SUGRA models were discussed in several early reports on supersymmetric signatures [78, 133, 134] and more intensely in the SUGRA Working Group Collaboration Report [135].

Sparticle spectrum: After the LEP data came out which showed that the extrapolation of gauge coupling constants was not consistent with a non-supersymmetric grand unification $SU(5)$ but was consistent with a supersymmetric one, we found it appropriate to compute the sparticle spectrum in a supergravity unified model using renormalization group evolution. This was done in [136] (see also [137]). Subsequent to the works of [136, 137] there were several analyses along these lines (see, e.g., [138, 139]). Currently such RG analyses are the standard procedure in generating the mass spectra in supergravity unified models.

Proton decay: It was pointed out by Weinberg [140] and by Sakai and Yanagida [141] that the supersymmetric grand unified models contain baryon and lepton number violating dimension five operators. Initial investigation of the main decay modes of the supersymmetric GUTs was done in [142, 143]. The first analysis within supergravity united model was carried out in [91, 92]. The supergravity analysis was extended in several further works: [138, 139, 144–151] (for the current status of proton decay vs experiment see [152, 153]).

Dark matter: Soon after the formulation of supergravity grand unification it was observed [154–156] that with R parity conservation that the neutralino could be a candidate for dark matter. Later it was shown by computation of the sparticle spectrum using RG evolution that under color and charge conservation that the neutralino was indeed the lightest supersymmetric particle and being neutral it was in fact a viable candidate for dark matter [136]. The precision computation of relic density using integration over the poles in the annihilation of neutralinos was given in [139, 157] using the technique used previously in the analysis of non-supersymmetric dark matter analyses [158]. Later on Dick and I worked on the direct detection of dark matter [114, 146, 159–162]. An analysis of the annual modulation effect on event rates in dark matter detectors was carried out in [163]. Dick continued the work on dark matter with other colleagues in later years (see e.g., [164]).

String inspired supergravity models: Since supergravity is the field point limit of strings, the string inspired supergravity models present an interesting class of high scale models for investigation. A number of models were investigated in [165–171]. Some of the phenomena investigated included $\mu \to e\gamma$ [172], charged lepton and neutrino masses and mixings [169], and Higgs boson phenomenology [170]. In [173] Yukawa couplings were computed for the model $CP^3 \times CP^2/Z_3 \times Z_3'$. In a later work

detecting physics in the post GUT and string scales was carried out [174].

Current status of SUGRA GUTs: The discovery of the Higgs boson at 126 GeV has given strong support for supergravity grand unification. Thus within the Standard Model vacuum stability is not guaranteed beyond scales of $Q \sim 10^{11}$ GeV while in supergravity grand unification one can allow for the stability of the vacuum up to GUT scales and beyond. Further, SUGRA GUT models predict the Higgs boson mass to lie below ~ 130 GeV [175] and it is quite remarkable that the observed value of the Higgs mass obeys this upper limit giving support to the idea of supergravity grand unification. The Higgs mass of $\sim$ 126 GeV leads to the average SUSY scale to lie in the TeV region which in part explains the non-observation of the sparticles thus far. Other virtues of the SUGRA GUT model include an explanation of how the electroweak symmetry breaks, i.e., it breaks via renormalization group effects. Thus it solves a major problem of the standard model where the Higgs mass is assumed to be tachyonic in an ad hoc fashion. It should be noted that historically SUGRA GUT provided the first hint that the top quark should be heavy, i.e., have a mass greater than ~ 100 GeV [104]. Further, in SUGRA GUT one can show that with charge and color conservation that the lightest supersymmetric particle is the neutralino over most of the parameter space of the model and thus the neutralino is a possible candidate for dark matter under the assumption of R party conservation. Detailed analyses show that the relic density of neutralinos consistent with the WMAP [176] and Planck [177] data can be gotten. Additionally, the current dark matter experiments are probing the parameter space of supergravity unified models in the neutralino$-$proton cross sections vs the neutralino mass plane and future detectors such as XENON1T [178] and LUX-ZEPLIN [179] can test a very significant part of the parameter space of SUGRA models. The measurement of the Higgs boson mass at 126 GeV points to a SUSY mass scale in the TeV region. Such a mass scale provides the desired loop correction needed to lift the tree level Higgs mass to the experimentally measured value. It helps suppress the flavor changing neutral current processes and also helps stabilize the proton against decay via B&L violating dimension five operators [180]. The RUN-II of the LHC will significantly expand the region of the parameter space of SUGRA models that will be probed [181]. It is hoped that the new data expected from LHC Run-II will provide us further evidence for the validity of supersymmetry and for SUGRA grand unification (For recent developments in SUGRA GUTs see, [182, 183]).

The memory of Richard Arnowitt will live on through his many contributions to physics. He will also live on in the memory of those who were privileged to know him. The Memorial Symposium at Texas A&M, College Station, September 19-20 was a fitting tribute to the life and times of Richard Arnowitt. Thanks to Marlan Scully and Roland Allen and the physics department at Texas A&M for organizing the Memorial Symposium.

ACKNOWLEDGMENTS

PN's research is supported in part by the NSF grant PHY-1314774.

[1] M. Gell-Mann, Physics **1**, 63 (1964).
[2] S. Weinberg, Phys. Rev. Lett. **17**, 616 (1966).
[3] D. Geffen, Phys. Rev. Letters 19, 770 (1967).
[4] B. Renner, Phys. Letters 21, 453 (1966).
[5] R. Arnowitt, M. H. Friedman and P. Nath, Phys. Rev. Lett. **19**, 1085 (1967).
[6] R. L. Arnowitt, M. H. Friedman and P. Nath, Phys. Rev. **174**, 1999 (1968).
[7] R. L. Arnowitt, M. H. Friedman and P. Nath, Phys. Rev. **174**, 2008 (1968).
[8] H. J. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967).
[9] J. S. Schwinger, Phys. Lett. B **24**, 473 (1967).
[10] J. Wess and B. Zumino, Phys. Rev. **163**, 1727 (1967).
[11] B. W. Lee and H. T. Nieh, Phys. Rev. **166**, 1507 (1968).
[12] S. G. Brown and G. B. West, Phys. Rev. Letters 19, 812 (1967).
[13] T. Das, V. S. Mathur and S. Okubo, Phys. Rev. Lett. **19**, 859 (1967).
[14] M. T. Vaughn, Phys. Rev. D **3**, 2241 (1971).
[15] R. L. Arnowitt, M. H. Friedman, P. Nath and R. Suitor, Phys. Rev. **175**, 1802 (1968).
[16] R. L. Arnowitt, M. H. Friedman, P. Nath and R. Suitor, Phys. Rev. D **3**, 594 (1971).
[17] R. L. Arnowitt, M. H. Friedman, P. Nath and R. Suitor, Phys. Rev. **175**, 1820 (1968).
[18] R. L. Arnowitt, M. H. Friedman and P. Nath, Nucl. Phys. B **10**, 578 (1969).
[19] R. L. Arnowitt, M. H. Friedman, P. Nath and R. Suitor, Phys. Rev. Lett. **26**, 104 (1971).
[20] R. L. Arnowitt, M. H. Friedman, P. Nath and P. Pond, Nuovo Cim. A **4**, 765 (1971).
[21] R. Arnowitt, P. Nath, P. Pond and M. H. Friedman, PRINT-69-690.
[22] T. D. Lee, S. Weinberg and B. Zumino, Phys. Rev. Lett. **18**, 1029 (1967).
[23] R. L. Arnowitt, M. H. Friedman and P. Nath, Phys. Rev. D **1**, 1813 (1970).
[24] M. Gell-Mann, R. J. Oakes and B. Renner, Phys. Rev. **175**, 2195 (1968).
[25] M. Veltman, Proc. Roy. Soc. A310, 107 (1967).
[26] R. Arnowitt, M.H. Friedman, PN, Phys.. Lett. 27 B, 657 (1968).
[27] J. S. Bell and R. Jackiw, Nuovo Cim. A **60**, 47 (1969).

[28] S. L. Adler, Phys. Rev. **177**, 2426 (1969).
[29] J. D. Bjorken, Phys. Rev. **179**, 1547 (1969).
[30] G. Mack, Nucl. Phys. B **5**, 499 (1968).
[31] H. A. Kastrup, Phys. Rev. **150**, 1183 (1966).
[32] K. G. Wilson, Phys. Rev. **179**, 1499 (1969).
[33] B. Zumino, *Brandeis Summer Institute Lectures, 1970* (MIT Press, Cambridge, Mass, 1970).
[34] P. Carruthers, Phys. Rept. **1**, 1 (1971).
[35] S. P. De Alwis and P. J. O'Donnell, Phys. Rev. D **2**, 1023 (1970).
[36] C. J. Isham, A. Salam and J. A. Strathdee, Phys. Rev. D **3**, 867 (1971).
[37] P. Nath, R. L. Arnowitt and M. H. Friedman, Phys. Rev. D **6**, 1572 (1972).
[38] M. H. Friedman, P. Nath and R. L. Arnowitt, Phys. Lett. B **42**, 361 (1972).
[39] R. L. Arnowitt, M. H. Friedman and P. Nath, Phys. Rev. D **7**, 1197 (1973).
[40] S.L. Glashow, in Hadrons and their interactions (Academic Press, New York, 1968), p 83.
[41] S. Weinberg, Phys. Rev. D **11**, 3583 (1975).
[42] G. 't Hooft, Nucl. Phys. B72, 461 (1974).
[43] A. De Rujula, H. H. Georgi, and S.L. Glashow, Phys. Rev. D. 12, 47 (1975).
[44] E. Witten, Nucl. Phys. B **156**, 269 (1979).
[45] P. Nath and R. L. Arnowitt, Phys. Rev. D **23**, 473 (1981).
[46] R. L. Arnowitt and P. Nath, Nucl. Phys. B **209**, 234 (1982).
[47] P. Nath and R. L. Arnowitt, Nucl. Phys. B **209**, 251 (1982).
[48] R. L. Arnowitt and P. Nath, Phys. Rev. D **25**, 595 (1982).
[49] C. Rosenzweig, J. Schechter and C. G. Trahern, Phys. Rev. D **21**, 3388 (1980).
[50] P. Di Vecchia and G. Veneziano, Nucl. Phys. B **171**, 253 (1980).
[51] E. Witten, Annals Phys. **128**, 363 (1980).
[52] P. Nath and R. L. Arnowitt, Phys. Lett. B **56**, 177 (1975).
[53] R. L. Arnowitt, P. Nath and B. Zumino, Phys. Lett. B **56**, 81 (1975).
[54] R. L. Arnowitt and P. Nath, Gen. Rel. Grav. **7**, 89 (1976).
[55] R. L. Arnowitt and P. Nath, Phys. Rev. D **15**, 1033 (1977).
[56] P. Nath and R. L. Arnowitt, Nucl. Phys. B **122**, 301 (1977).
[57] R. L. Arnowitt and P. Nath, Nucl. Phys. B **144**, 363 (1978).
[58] P. Nath and R. L. Arnowitt, Phys. Rev. Lett. **42**, 138 (1979).
[59] R. L. Arnowitt and P. Nath, Phys. Rev. D **18**, 2759 (1978).
[60] R. L. Arnowitt and P. Nath, Nucl. Phys. B **161**, 321 (1979).
[61] R. L. Arnowitt and P. Nath, Phys. Rev. Lett. **44**, 223 (1980).
[62] D. Z. Freedman, P. van Nieuwenhuizen and S. Ferrara, Phys. Rev. D **13**, 3214 (1976).
[63] S. Deser and B. Zumino, Phys. Lett. B **62**, 335 (1976).
[64] P. Van Nieuwenhuizen, Phys. Rept. **68**, 189 (1981).
[65] P. Nath and R. L. Arnowitt, Phys. Lett. B **65**, 73 (1976).
[66] R. L. Arnowitt and P. Nath, Phys. Lett. B **78**, 581 (1978).
[67] P. Nath and R. L. Arnowitt, Nucl. Phys. B **165**, 462 (1980).
[68] P. Breitenlohner, Phys. Lett. B **67**, 49 (1977).
[69] J. Wess and B. Zumino, Phys. Lett. B **66**, 361 (1977).
[70] L. Brink, M. Gell-Mann, P. Ramond and J. H. Schwarz, Phys. Lett. B **74**, 336 (1978).
[71] L. Brink, M. Gell-Mann, P. Ramond and J. H. Schwarz, Phys. Lett. B **76**, 417 (1978).
[72] A. H. Chamseddine, hep-th/0601146.
[73] Ali H. Chamseddine, Supersymmetry and Higher Spin Fields, *PH. D. Thesis, Chapters IV and V, Imperial College, London University* (1976).
[74] A. H. Chamseddine and P. C. West, Nucl. Phys. B **129**, 39 (1977).
[75] A. H. Chamseddine, Nucl. Phys. B **185**, 403 (1981).
[76] A. H. Chamseddine, Phys. Rev. D **24**, 3065 (1981).
[77] E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and P. van Nieuwenhuizen, Nucl. Phys. B **147**, 105 (1979).
[78] P. Nath, R. L. Arnowitt and A. H. Chamseddine, NUB-2613.
[79] A. H. Chamseddine, R. L. Arnowitt and P. Nath, Phys. Rev. Lett. **49**, 970 (1982).
[80] P. Nath, R. L. Arnowitt and A. H. Chamseddine, Phys. Lett. B **121**, 33 (1983).
[81] P. Nath, R. L. Arnowitt and A. H. Chamseddine, Nucl. Phys. B **227**, 121 (1983).
[82] E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Phys. Lett. B **116**, 231 (1982).
[83] E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Nucl. Phys. B **212**, 413 (1983).
[84] J. Bagger and E. Witten, Phys. Lett. B **118**, 103 (1982).
[85] S. Weinberg, Phys. Rev. Lett. **48**, 1776 (1982).
[86] L. J. Hall, J. D. Lykken and S. Weinberg, Phys. Rev. D **27**, 2359 (1983).
[87] R. L. Arnowitt, A. H. Chamseddine and P. Nath, Phys. Rev. Lett. **50**, 232 (1983).
[88] A. H. Chamseddine, P. Nath and R. L. Arnowitt, Phys. Lett. B **129**, 445 (1983) [Erratum-ibid. B **132**, 467 (1983)].
[89] P. Nath, R. L. Arnowitt and A. H. Chamseddine, HUTP-83/A077, NUB-2588a.
[90] T. C. Yuan, R. L. Arnowitt, A. H. Chamseddine and P. Nath, Z. Phys. C **26**, 407 (1984).
[91] R. L. Arnowitt, A. H. Chamseddine and P. Nath, Phys. Lett. B **156**, 215 (1985).
[92] P. Nath, A. H. Chamseddine and R. L. Arnowitt, Phys. Rev. D **32**, 2348 (1985).
[93] A. H. Chamseddine, P. Nath and R. L. Arnowitt, Phys. Lett. B **174**, 399 (1986).
[94] R. Arnowitt, A. H. Chamseddine and P. Nath, Int. J. Mod. Phys. A **27**, 1230028 (2012) [Erratum-ibid. A **27**, 1292009 (2012)] [arXiv:1206.3175 [physics.hist-ph]].
[95] A. H. Chamseddine, R. L. Arnowitt and P. Nath, Nucl. Phys. Proc. Suppl. **101**, 145 (2001) [hep-ph/0102286].
[96] R. Barbieri, S. Ferrara and C. A. Savoy, Phys. Lett. B **119**, 343 (1982).
[97] L. E. Ibanez, Phys. Lett. B **118**, 73 (1982).
[98] H. P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B **124**, 337 (1983).
[99] J. R. Ellis, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B **121**, 123 (1983).
[100] S. Weinberg, Phys. Rev. Lett. **50**, 387 (1983).

[101] L. E. Ibanez and C. Lopez, Phys. Lett. B **126**, 54 (1983).

[102] L. E. Ibanez and C. Lopez, Nucl. Phys. B **233**, 511 (1984).

[103] L. E. Ibanez, C. Lopez and C. Munoz, Nucl. Phys. B **256**, 218 (1985).

[104] L. Alvarez-Gaume, J. Polchinski and M. B. Wise, Nucl. Phys. B **221**, 495 (1983).

[105] S. K. Soni and H. A. Weldon, Phys. Lett. B **126**, 215 (1983).

[106] R. L. Arnowitt and P. Nath, Phys. Rev. D **46**, 3981 (1992).

[107] S. P. Martin and M. T. Vaughn, Phys. Rev. D **50**, 2282 (1994) [Erratum-ibid. D **78**, 039903 (2008)] [hep-ph/9311340].

[108] L. E. Ibanez and G. G. Ross, Comptes Rendus Physique **8**, 1013 (2007) [hep-ph/0702046 [HEP-PH]].

[109] J. M. Frere, D. R. T. Jones and S. Raby, Nucl. Phys. B **222**, 11 (1983).

[110] M. Claudson, L. J. Hall and I. Hinchliffe, Nucl. Phys. B **228**, 501 (1983).

[111] M. Drees, M. Gluck and K. Grassie, Phys. Lett. B **157**, 164 (1985).

[112] M. Drees, Phys. Lett. B **158**, 409 (1985).

[113] J. R. Ellis, K. Enqvist, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B **155**, 381 (1985).

[114] P. Nath and R. L. Arnowitt, Phys. Rev. D **56**, 2820 (1997) [hep-ph/9701301].

[115] D. A. Dicus, S. Nandi, W. W. Repko and X. Tata, Phys. Rev. Lett. **51**, 1030 (1983) [Erratum-ibid. **51**, 1813 (1983)].

[116] D. A. Dicus, S. Nandi and X. Tata, Phys. Lett. B **129**, 451 (1983) [Erratum-ibid. B **145**, 448 (1984)].

[117] J. M. Frere and G. L. Kane, Nucl. Phys. B **223**, 331 (1983).

[118] J. R. Ellis, J. M. Frere, J. S. Hagelin, G. L. Kane and S. T. Petcov, Phys. Lett. B **132**, 436 (1983).

[119] J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos and M. Srednicki, Phys. Lett. B **127**, 233 (1983).

[120] J. Polchinski and M. B. Wise, Phys. Lett. B **125**, 393 (1983).

[121] B. Grinstein, J. Polchinski and M. B. Wise, Phys. Lett. B **130**, 285 (1983).

[122] M. Claudson, L. J. Hall and I. Hinchliffe, Phys. Lett. B **130**, 260 (1983).

[123] D. A. Kosower, L. M. Krauss and N. Sakai, Phys. Lett. B **133**, 305 (1983).

[124] G. W. Bennett *et al.* [Muon G-2 Collaboration], Phys. Rev. D **73**, 072003 (2006) [hep-ex/0602035].

[125] K. Hagiwara, R. Liao, A. D. Martin, D. Nomura and T. Teubner, J. Phys. G **38**, 085003 (2011) [arXiv:1105.3149 [hep-ph]].

[126] M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, Eur. Phys. J. C **71**, 1515 (2011) [Erratum-ibid. C **72**, 1874 (2012)] [arXiv:1010.4180 [hep-ph]].

[127] H. Baer, K. Hagiwara and X. Tata, Phys. Rev. Lett. **57**, 294 (1986).

[128] P. Nath and R. L. Arnowitt, Mod. Phys. Lett. A **2**, 331 (1987).

[129] R. L. Arnowitt, R. M. Barnett, P. Nath and F. Paige, Int. J. Mod. Phys. A **2**, 1113 (1987).

[130] H. Baer, C. h. Chen, F. Paige and X. Tata, Phys. Rev. D **53**, 6241 (1996) [hep-ph/9512383].

[131] V. D. Barger, C. Kao and T. j. Li, Phys. Lett. B **433**, 328 (1998) [hep-ph/9804451].

[132] E. Accomando, R. L. Arnowitt and B. Dutta, Phys. Lett. B **475**, 176 (2000) [hep-ph/9811300].

[133] H. P. Nilles, Phys. Rept. **110**, 1 (1984).

[134] H. E. Haber and G. L. Kane, Phys. Rept. **117**, 75 (1985).

[135] V. Barger *et al.* [SUGRA Working Group Collaboration], hep-ph/0003154.

[136] R. L. Arnowitt and P. Nath, Phys. Rev. Lett. **69**, 725 (1992).

[137] G. G. Ross and R. G. Roberts, Nucl. Phys. B **377**, 571 (1992).

[138] P. Nath and R. L. Arnowitt, Phys. Lett. B **287**, 89 (1992).

[139] P. Nath and R. L. Arnowitt, Phys. Rev. Lett. **70**, 3696 (1993) [hep-ph/9302318].

[140] S. Weinberg, Phys. Rev. D **26**, 287 (1982).

[141] N. Sakai and T. Yanagida, Nucl. Phys. B **197**, 533 (1982).

[142] S. Dimopoulos, S. Raby and F. Wilczek, Phys. Lett. B **112**, 133 (1982).

[143] J. R. Ellis, D. V. Nanopoulos and S. Rudaz, Nucl. Phys. B **202**, 43 (1982).

[144] P. Nath and R. L. Arnowitt, Phys. Rev. D **38**, 1479 (1988).

[145] R. L. Arnowitt and P. Nath, Phys. Rev. Lett. **62**, 2225 (1989).

[146] R. L. Arnowitt and P. Nath, Phys. Lett. B **437**, 344 (1998) [hep-ph/9801246].

[147] P. Nath and R. L. Arnowitt, Phys. Atom. Nucl. **63**, 1151 (2000) [Yad. Fiz. **63**, 1227 (2000)].

[148] R. L. Arnowitt and P. Nath, Phys. Rev. D **49**, 1479 (1994) [hep-ph/9309252].

[149] P. Nath and R. L. Arnowitt, Phys. Atom. Nucl. **61**, 975 (1998) [Yad. Fiz. **61**, 1069 (1998)] [hep-ph/9708469].

[150] R. L. Arnowitt and P. Nath, Annals N. Y. Acad. Sci. **518**, 337 (1987).

[151] R. L. Arnowitt and P. Nath, Phys. Rev. D **36**, 3423 (1987).

[152] P. Nath and P. Fileviez Perez, Phys. Rept. **441**, 191 (2007) [hep-ph/0601023].

[153] K. S. Babu, E. Kearns, U. Al-Binni, S. Banerjee, D. V. Baxter, Z. Berezhiani, M. Bergevin and S. Bhattacharya *et al.*, arXiv:1311.5285 [hep-ph].

[154] H. Goldberg, Phys. Rev. Lett. **50**, 1419 (1983) [Erratum-ibid. **103**, 099905 (2009)].

[155] L. M. Krauss, Nucl. Phys. B **227**, 556 (1983).

[156] J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos, K. A. Olive and M. Srednicki, Nucl. Phys. B **238**, 453 (1984).

[157] R. L. Arnowitt and P. Nath, Phys. Lett. B **299**, 58 (1993) [Erratum-ibid. B **307**, 403 (1993)] [hep-ph/9302317].

[158] K. Griest and D. Seckel, Phys. Rev. D **43**, 3191 (1991).

[159] R. L. Arnowitt and P. Nath, Mod. Phys. Lett. A **10**, 1257 (1995) [hep-ph/9408226].

[160] P. Nath and R. L. Arnowitt, Phys. Rev. Lett. **74**, 4592 (1995) [hep-ph/9409301].

[161] P. Nath and R. L. Arnowitt, Phys. Lett. B **336**, 395 (1994) [hep-ph/9406389].

[162] R. L. Arnowitt and P. Nath, Phys. Rev. D **54**, 2374 (1996) [hep-ph/9509260].

[163] R. L. Arnowitt and P. Nath, Phys. Rev. D **60**, 044002 (1999) [hep-ph/9902237].

[164] R. L. Arnowitt, B. Dutta, A. Gurrola, T. Kamon, A. Krislock and D. Toback, Phys. Rev. Lett. **100**,

231802 (2008) [arXiv:0802.2968 [hep-ph]].
[165] P. Nath and R. L. Arnowitt, Phys. Rev. D **39**, 2006 (1989).
[166] R. L. Arnowitt and P. Nath, Phys. Rev. D **40**, 191 (1989).
[167] P. Nath and R. L. Arnowitt, Phys. Rev. Lett. **62**, 1437 (1989).
[168] P. Nath and R. L. Arnowitt, Phys. Rev. D **42**, 2948 (1990).
[169] R. L. Arnowitt and P. Nath, Phys. Lett. B **244**, 203 (1990).
[170] P. Nath and R. L. Arnowitt, Phys. Rev. D **43**, 3739 (1991).
[171] R. L. Arnowitt and P. Nath, Phys. Rev. Lett. **60**, 1817 (1988).
[172] R. L. Arnowitt and P. Nath, Phys. Rev. Lett. **66**, 2708 (1991).
[173] J. z. Wu, R. L. Arnowitt and P. Nath, Int. J. Mod. Phys. A **6**, 381 (1991).
[174] R. L. Arnowitt and P. Nath, Phys. Rev. D **56**, 2833 (1997) [hep-ph/9701325].
[175] S. Akula, B. Altunkaynak, D. Feldman, P. Nath and G. Peim, Phys. Rev. D **85**, 075001 (2012) [arXiv:1112.3645 [hep-ph]].
[176] G. Hinshaw *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **208**, 19 (2013) [arXiv:1212.5226 [astro-ph.CO]].
[177] P. A. R. Ade *et al.* [Planck Collaboration], Astron. Astrophys. **571**, A1 (2014) [arXiv:1303.5062 [astro-ph.CO]].
[178] E. Aprile [XENON1T Collaboration], Springer Proc. Phys. **148**, 93 (2013) [arXiv:1206.6288 [astro-ph.IM]].
[179] C. Ghag, arXiv:1412.4660 [hep-ex].
[180] M. Liu and P. Nath, Phys. Rev. D **87**, no. 9, 095012 (2013) [arXiv:1303.7472 [hep-ph]].
[181] H. Baer, V. Barger, A. Lessa and X. Tata, JHEP **0909**, 063 (2009) [arXiv:0907.1922 [hep-ph]].
[182] D. Francescone, S. Akula, B. Altunkaynak and P. Nath, arXiv:1410.4999 [hep-ph].
[183] P. Nath, arXiv:1501.01679 [hep-ph].